# 8. THE UNDECIDABLE

## §8.1. Axiomatic Systems

As I've said, mathematical truth is established by logic, starting with some fundamental assumptions called axioms. One is obliged to accept the conclusions provided one accepts the logical principles used as well as the axioms. There is a real sense in which a set of axioms is a creed, like a religious creed.

Euclid is credited with devising the first set of axioms – the axioms for Geometry or, as we now consider it, the axioms for Euclidean Geometry. These axioms were considered to be 'self-evident'. Axioms such as "between any two distinct points there is exactly one straight line". Far from being self-evident, this is based on experimental evidence and has the same status as a scientific 'fact'.

Axioms for other mathematical systems were proposed in the late 19th century. The first were the axioms of group theory. Never mind what group theory is or what the axioms are. Rather than self-evident truths they were considered to simply make up a definition of a group.

These days there's much controversy about gay marriage. Some regard it as self-evident that 'marriage' means an arrangement between a man and a woman. In fact, it's merely the definition of the word 'marriage'. Certainly there's no doubt that this is what was implied by the word over centuries. Others say the definition should be broadened. There's a long history of the meaning of words being broadened.

'Money' once referred to what we now call 'currency' – coins and notes, but the meaning has been broadened to include electronic transactions. That doesn't mean that the meaning of 'marriage' *should* be broadened. There are strong arguments on both sides. The point I'm making is that each person who uses the word 'marriage' should be prepared to state their definition.

The attitude towards Euclid's axioms changed in the eighteenth century. They were no longer considered to be self-evident, but merely part of the definition of a particular geometry called Euclidean geometry. Other, slightly different, sets of axioms were set up for other geometries. From a mathematical point of view all of them are correct. It's up to the scientist, the physicist, the cosmologist, to decide which is correct for our universe. And the jury is still out on that question.

A rather different state of affairs exists for Set Theory. A 'set' is a collection of 'things'. In Axiomatic Set Theory these things are mathematical objects. Now unlike Group Theory, where there are lots of systems satisfying the axioms, in axiomatic set theory we are attempting to describe a concept that we hold intuitively.

## §8.2. The Russell Paradox

Set theory has come to underlie all of mathematics, so in a sense it is the foundation for all mathematics. Up to the end of the 19th century it was considered that the truths of set theory were self-evident, just as we don't fuss too much about the logic we employ. One of the assumptions is that for any property that things might have there is a corresponding set, consisting of all the things that have that property. This is the process of turning an adjective into a noun. 'Black' is an adjective, so there is the set of all black things. But the philosopher Bertrand Russell, who was interested in the foundations of mathematics, pointed out that the set of all sets that do not belong to themselves is self-contradictory.

Perhaps a bit of notation will help us to understand this. The fundamental property of sets is that things belong to them. We denote the fact that the thing $x$ belongs to the set S by the notation $x \in \mathrm{S}$.

If P is a property, like being black, and $x$ is a thing, we denote the statement that $x$ has the property P by P$x$. So if $c$ = a crow and B$x$ = "$x$ is black" then B$c$ is a true statement, while B$s$ is false if $x$ = a dove. Crows are black but doves are not.

The set that corresponds to the property P is denoted by $\{x \mid Px\}$. Read it as "the set of all $x$ such that P$x$ (or P$x$ is true). The naïve assumption was that for all properties *P* there must be a set $\{x \mid Px\}$.

Russell considered the property of something not belonging to itself – in the sense of set belonging. Here the something is a set. A set can belong to another set because it is possible to have sets of sets, or sets of sets of sets ....

If T is the set of all pairs of distinct whole numbers then the set {3, 5}, consisting of just 3 and 5, would belong to T.

The symbol for "not belonging" is $\notin$, just like the symbol for "not equals" is obtained by crossing out the equals sign, as in $\neq$. Now some sets clearly don't belong to themselves. The set of all positive numbers is not a positive number. But the set of all sets is a set.

So Russell said, what if $S = \{x \mid x \notin x\}$? This would be the set of all sets that are not members of themselves. This would be the case for most sets we might think of.

The set of all even numbers is not an even number. The set of all triangles is not a triangle. The set of all infinite sets is an infinite set.

The question is:

**Does S belong to S?**

Clearly the answer would have to be either “yes” or “no”, but let’s consider each possibility in turn.

**SUPPOSE that S $\in$ S.**
Then it must satisfy the corresponding property, that is S $\notin$ S. This is a contradiction.

**SUPPOSE that S $\notin$ S.**
Then S satisfies the property that defines S and so S $\in$ S. Again, a contradiction.

This seems like one of those logical paradoxes like the sentence “THIS SENTENCE IS FALSE”. But we can’t ignore it. Under our naïve concept of set theory such a set exists. If we want to ban it from being a set we’d better explain to it why it’s being kicked out!

This may also remind you of the argument from the chapter on the uncountable. The difference is that in that case there was an assumption that led to the contradiction. If one can find a different chairman for every committee then we get a contradiction. Therefore it’s impossible to provide a different chairman for every committee. It is false that there is the same number of subsets as elements.

But with the Russell Paradox there appears to be no such initial assumption, apart from the intuitively obvious ‘fact’ that for every property there’s a set of all things with that property. Well, then, intuitively obvious or not, this assumption has to go.

Here we have a fundamental contradiction in set theory. And since we want to build mathematics upon set theory, all of mathematics would fall to the ground if we didn’t remove such a flaw. If you allow a single contradiction into mathematics you can prove anything.

I remember one of my lecturers telling me this and when someone asked him to prove that he was the Pope, assuming that 0 = 1, he said, “If 0 = 1 then, adding 1 to both sides, then 1 = 2. The Pope and I are two people, so the Pope and I are the one person. QED.”

Well, you can imagine the fuss that Russell’s Paradox caused when it was first announced. At least it caused a fuss amongst those who were bothered about the foundations of mathematics.

Ordinary working mathematicians just said, “oh, that’s interesting” and went back to their work. They knew that someone would fix up the problem, and that they did.

The way of fixing up the problem was to set up a collection of axioms that made some restrictions on which properties *do* lead to a set. There have been several formulations but they have all been proved to be equivalent to one another. The most well-known set of axioms are called the ZF axioms, after their proposers Zermelo and Fraenkel. I won't list them here because they're long and sound quite technical. Basically they mostly say that "if such and such is a set the so and so is a set". They are all dependent on already having some sets with which to make other sets - except for the first axiom, the existence of the **empty** set.

The empty set is the set with no elements. It doesn't matter what the no elements are. The set of unicorns is the same empty set as the set of elves or the set of whole numbers lying strictly between 1 and 2. Axiom 1 in the ZF system says: There exists a set corresponding to the property $x \neq x$, that is $\{x \mid x \neq x\}$ exists. The symbol for the empty set is $\varnothing$. Now you might be thinking that is silly to have a set with nothing in it.

"Oh, I have a collection of vintage Rolls Royce automobiles."
"Wow! How many have you got?"
"Oh, it's the empty set."

Stupid as it might seem, where would we be without the number zero? For centuries zero was never considered a number. Why have a number for something that doesn't exist. Yet, our modern system of notation for numbers relies on zero. The difference between my bank balance and that of Bill Gates is just a whole lot of zeros!

Now there's something rather delightful in the fact that all of mathematics can be manufactured from the empty set. First there's the set {∅} that contains just the empty set. It isn't the empty set itself because it *does* have something in it, even though that something is empty. Then there is {∅, {∅}}.

This set contains two sets, the empty set itself, and the set consisting of the empty set. It might seem that we're splitting hairs here, but the distinction between ∅ and {∅} is important. In fact, when the number 2 is defined it is defined in this way of developing mathematics, it is the set {∅, {∅}} and 3 is {∅, {∅}, {∅, {∅}}.

If this seems a rather esoteric way of defining the number 3, let me ask how you would define it. I'm sure what you might come up with would be more intelligible to a typical kindergarten pupil than {∅, {∅}, {∅, {∅}} but it wouldn't stand up to the high standard of rigour that professional mathematicians require.

You might say that this shows that God created mathematics. Just as God created the world from a void he created the whole of mathematics from the empty set! On the other hand, if you are somewhat of an atheist, at least you'll find a resonance between mathematics being created from the empty set and the big-bang theory of how the universe began.

## §8.3. Axioms for Mathematics

Almost all of mathematics can be built up from the following axioms. They are called the **Zermelo-Fraenkel Axioms**, or **ZF** for short. Other foundations have been suggested, but they are all equivalent to the ZF creed. For ‘creed’ it is – just as a religious creed. They are statements whose truths are taken without proof. One just has to believe in them. Remember that it is not possible to prove something from nothing.


In addition, there are assumptions about logic, we would be considering logical axioms as well. These will regulate the use of words such as ‘and’, ‘or’ and ‘implies’.

Six of the eight ZF axioms are:

**Equality:** Two sets are equal if they have precisely the same elements.

**Empty Set:** There is a set with no elements.

**Pairs:** If S, T are sets there is a set with just S and T as elements.
**Powers:** If S is a set so is the set of all subsets of S.

**Union:** If S is a set so is the set of all elements of elements of S.

**Specification:** If S is a set and P is any property that can be expressed entirely in terms of set membership, then there is a set whose elements are precisely those elements of S for which the property holds.

The other two axioms are a bit more technical, so I'll omit them. A full discussion can be found in my notes on *Set Theory*. On the basis of these eight axioms virtually the whole of mathematics can be built. (This is outlined in my *Set Theory* Notes.)

So can we now be assured that no further contradiction, like Russell's Paradox will arise? This amounts to asking whether the ZF axioms are consistent. The slightly disturbing answer is that no, we do not know that they are consistent. Most mathematicians believe that they are, but most mathematicians believe that we will never be able to prove consistency.

## §8.4. Consistency

A set of axioms is **inconsistent** if a contradiction can be validly derived from them. If it is not inconsistent then it is defined to be **consistent**. The easiest way to prove consistency is to come up with a model for the axioms, that is, an actual interpretation that satisfies all the axioms.

It's easy to come up with an inconsistent set of axioms. For example consider the following axioms for a *super number*. The set of super numbers has two operations, called addition and multiplication, such that the following axioms hold.

**Axiom 1:** There's a super number 0, such that $n + 0 = n$ for all super numbers, $n$.

**Axiom 2:** There is a super number 1 such that $1 + 1 \neq 1$.

**Axiom 3:**
$(x + y)z = xy + xz$ for all super numbers $x$, $y$ and $z$.

**Axiom 4:** There's a super number $\infty$ such that $0\infty = 1$.

This system of axioms is inconsistent. Here's a proof.

By axiom 1: $0 + 0 = 0$, and so $(0 + 0)\infty = 0\infty$.
By axiom 3: $0\infty + 0\infty = 0\infty$.
By axiom 4: $1 + 1 = 1$, contradicting axiom 2.

Here's another rather exotic axiomatic system that I've constructed to illustrate the concept of consistency. I call the system a **society**. In a society there's a set of undefined things called **persons** and three undefined relations:

father of,
mother of,
married to

Now the terminology suggests we're thinking of family relationships, and certainly that's what inspired these axioms. But it must be emphasized that these things called 'persons' are to be considered as undefined and so we must not make any use of what we know of actual family relationships.

We assume the following axioms:

**Axiom 1:** There exists a person.

**Axiom 2:** Each person has a unique mother and a unique father.

**Axiom 3:** If two people have the same mother then they have the same father.

**Axiom 4:** The mother and father of every person must be married.

**Axiom 5:** If two people have the same father they can't marry.

You will probably question whether these axioms capture the complexities of modern family life, but that's not the question.

I'd like to define a **parent** to be a 'person' who's either a mother or a father and a **grandmother** to be the mother of a parent.

**Theorem 1:** Every person has exactly two grandmothers.
**Proof:** Let Peter be a person.
By Axiom 2 Peter has exactly one father, who we'll call Frank, and exactly one mother, called Michelle.
By Axiom 4, Frank is married to Michelle.
Suppose Frank = Michelle.
Then by axiom 4, Frank is married to himself, contradicting Axiom 5. Hence Frank ≠ Michelle.

By Axiom 2, Frank has exactly one mother, denoted by Mildred and Michelle has exactly one mother, denoted by Mary.

Suppose Mildred = Mary. That is, suppose Frank has the same mother as Michelle. Then by Axiom 3 Frank and Michelle have the same father, denoted by Phillip.

By Axiom 5, Frank and Michelle can't marry, contradicting what we proved earlier.

Hence Mildred ≠ Mary and so Peter has exactly two grandmothers.

Notice that I proved the theorem only using the axioms, and without appealing to my intuition, or knowledge of society. Now are these axioms consistent? There's no point in proving theorems for a non-existent system. To do this we need to devise a model – a concrete example in which these axioms hold.

Here's a model for this system. A 'person' is one of the positive integers 1, 2, 3, … The father of $n$ is $2n$ and the mother of $n$ is $2n + 1$. Person $m$ is married to person $n$ if $m + n$ is odd. This system is a society.

Now this model is quite different to the one I might have had in mind when constructing these axioms. For a start it allows polygamy on a grand scale. Since odd + even is odd, person 2 (and any even person for that matter) is married to every odd person. This is indeed an odd model! But let's check the axioms.

**Axiom 1** and **Axiom 2** are clearly true for this model.

**Axiom 3:** If $m$ and $n$ have the same mother then $2n + 1 = 2m + 1$ and so $2n = 2m$, which means that they have the same father.

**Axiom 4:** The father and mother of person $n$ are $2n$ and $2n + 1$ respectively. Since their sum is odd, they are married.

**Axiom 5:** If $m$ and $n$ have the same father the $2m = 2n$ and so $m = n$. Thus $m + n$ is even and so they can't be married.

The fact that a model exists for a society, means that the axioms are consistent. But societies as described by these axioms can be very different to the model I had in mind when I devised the axioms.

In Axiomatic Set Theory we often consider extra 'optional axioms'. We could add optional axioms to make it more like the society of people and their families. But we would have to be very flexible, because there some rather strange family relationships in today's society.

## I'M MY OWN GRANDPA!

Many, many years ago when I was twenty-three
I was married to a widow who was as pretty as could be
This widow had a grown-up daughter who had hair of red
My father fell in love with her and soon they too were wed.

This made my dad my son-in-law and really changed my life.
For now my daughter was my mother, 'cause she was my father's wife.
And to complicate the matter, even though it brought me joy
I soon became the father of a bouncing baby boy.

My little baby boy then became a brother-in-law to Dad
And so became my uncle, though it made me very sad,
For if he were my uncle, then that also made him brother
Of the widow's grown-up daughter, who was of course my step-mother.

Father's wife then had a son who kept them on the run,
And he became my grandchild, for he was my daughter's son.
My wife is now my mother's mother and it makes me blue
Because although she is my wife, she's my grandmother too.

Now if my wife is my grandmother, then I'm her grandchild,
And every time I think of it, it nearly drives me wild.
'Cause now I have become the strangest case you ever saw
As husband of my grandmother, I'm my own grandpa.

I'm my own grandpa
It sounds funny, I know but it is really so
I'm my own grandpa.

***Written by Latham Dwight and Jeff Moe and published by Colgems-EMI Music.***

## §8.5. The Axiom of Choice

Now, what's really interesting is that there a few things that can't be proved from the ZF axioms which most mathematicians believe are true. One of these is the Axiom of Choice, abbreviated to AXC. In a nutshell the AXC says that if you have a whole bunch of non-empty sets you can simultaneously choose one thing out of each of them. This seems an obvious enough statement but, remember that it says that this is possible, even if the sets are infinite and even if there are infinitely many of them.

Of course such a choice is impossible in practice because it would take infinite time, but we're not talking about 'in practice'. The question is, does such a choice exist and can they choices form a set? (The last question is not quite the one that is asked, but it's near enough for our purposes.)

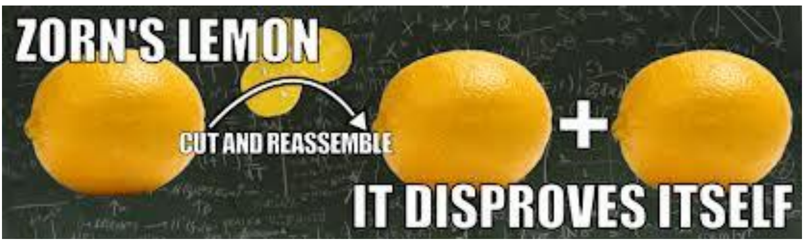
The Axiom of Choice has been proved to be **consistent with**, and **independent of**, the ZF axioms. To show this you assume the ZF axioms and construct a model in which not only the ZF axioms hold, but also the Axiom of Choice. That's the 'consistent with' part. Then you construct a different model, with a different definition of 'belonging to' that satisfies the ZF axioms but does *not* satisfy the Axiom of Choice. That's the 'independent of' half of the statement. Putting these halves together we come up with the statement:

**THE AXIOM OF CHOICE IS UNDECIDABLE.**

This means that, assuming the ZF axioms are consistent, you'll never be able to prove that the AXC is true. But nor will you ever be able to prove that it's false. If ever a contradiction arises in mathematics when using the Axiom of Choice it won't be the fault of that axiom. It will mean that an inconsistency will have been found in the ZF axioms themselves. If ever a contradiction arises from denying the Axiom of Choice it will mean that the ZF axioms themselves are inconsistent, not the denial itself.

The bottom line is that you are free to choose! You can believe in AXC or not. Both positions are logically valid. Naturally, like most mathematicians, you will no doubt opt to believe in AXC. It sounds so plausible. But before you become a paid-up member of the Axiom of Choice religion, let me point out the following consequence of the Axiom of Choice.


It has been proved, assuming the ZF axioms, together with the AXC, that in principle it's possible to take a solid ball and dissect it into several pieces and to reassemble the pieces to make *two* solid balls *of the same size as the original one*!

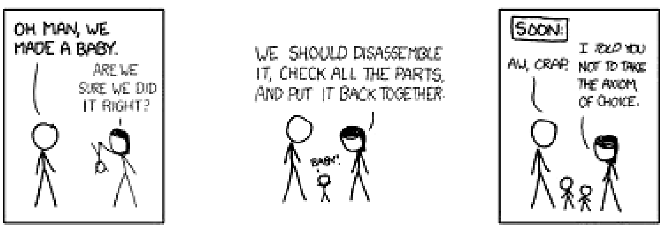



Your reaction to this is probably to say that this proves that the AXC is false. After all, such a situation would contradict the law of conservation of volume, surely. If you take a piece of wood its volume would remain constant no matter how you cut it up and reassembled the pieces. That is, ignoring the sawdust which, of course, we're doing.

However the law of conservation of volume only applies if the pieces have a defined volume. If a set of points is highly fragmented, like a cloud of infinitely small particles, then it's not possible to define its volume.

The way of dissecting the original sphere and reassembling them is not something one could replicate, even with precision tools. If it was possible to convert one ounce of gold into two with a laser cutter, the price of gold would plummet! The 'pieces' that are required to perform this magic are so highly fragmented that their volumes don't exist.

Needless to say, while most mathematicians are happy to accept the Axiom of Choice, because it simplifies the statements of many of their theorems, there's a determined minority who reject it. A comforting thought is that no bridge will ever fall down because its engineer believed or didn't believe in the AXC.



The difference between believing or not believing is more aesthetic than practical. In this sense it's rather different to a religious belief. The Axiom of Choice believers will never wage war on the infidels, and no mathematician will become a martyr to his or her belief. The general consensus is that one should try not to use the Axiom of Choice, but if necessary one uses it, and admits that it is "on the basis of the Axiom of Choice".

## §8.6. The Peano Axioms

The very first mathematical system we ever encountered was the system of the natural numbers 1, 2, 3, … When we did so, in kindergarten or even before, we were not interested in precise definitions. We learnt the many properties of natural numbers on the authority of our parents and teachers.

Nowhere did we see a definition of the number 2, or a precise proof of the fact that 2 + 2 = 4. We might have experimented with a few pairs of objects and observed that combining one with another we got a collection which, when we counted, gave us 4. Hence we learnt our mathematics as an experimental science.

Of course we did notice that sometimes it didn't work. Pour a litre of water into a bowl containing a litre of sugar and you'll find you get a whole lot less than a litre of sugar syrup. This can be explained, in part, by the air spaces between the grains of sugar, but to account for the reduction in volume completely you need to take the chemistry of solutions into account. Nevertheless you understood that something different is going on here and that $1 + 1 = 2$ is still valid mathematically.

One approach to constructing the natural numbers, and their arithmetic, rigorously is to build them up as sets of sets of sets within axiomatic set theory. Another approach is to define them by a set of axioms, the Peano Axioms.

We postulate a set of undefined things, together with an undefined function **successor**. You can think of the successor of $n$ as $n + 1$, written $\boldsymbol{n}^+$, However that interpretation isn't specifically part of the axioms and, moreover, we need to define addition and then prove that $n^+ = n + 1$ from the axioms.

**Axiom 1:** 0 is a natural number.

**Axiom 2:** If $n$ is a natural number then so is $n^+$.

**Axiom 3:** There's no natural number $n$ for which $n^+ = 0$.
**Axiom 4:** If S is any set of natural numbers that contains 0, and contains $n^+$ whenever it contains $n$, then S is the set of all natural numbers.

On the basis of these axioms we can define addition and multiplication and prove the basic properties of arithmetic.

## §8.7. Gödel’s Incompleteness Theorem

We’ve seen how mathematical systems, such as set theory, can be built up on the basis of a set of axioms. Provided that a set of axioms is consistent we can prove meaningful theorems about the system. But can we prove every true statement from the axioms?

If we left out one of the set theory axioms there would be true statements about arithmetic that couldn’t be proved. A set of axioms is complete if every true statement about the system can be proved. Are the ZF axioms complete?

The answer is no. Well, then, we’d better add some extra axioms to make it complete. Unfortunately that’s not possible.

In 1931 Kurt Gödel proved that, not only are the ZF axioms incomplete. It’s not possible for a finite set of axioms to exist for any formal system in which basic arithmetic can be formulated, such that the axioms are complete.

He did this by converting every statement in such a system into an arithmetic statement. He managed to express to express the statement “this statement cannot be proved from the axioms” as a statement about arithmetic. Such a self-referential statement cannot be proved from the axioms, yet it’s a true statement and corresponds to a true statement about arithmetic.

Gödel’s original proof is very long, and very hard to read. A much simpler proof by Nagel & Newman in

2001 converts the statement to one about computability, and uses the machinery of Turing Machine to show that completeness would imply that the halting problem could be solved, which we know is impossible.

So here we are left with this unsatisfactory state of affairs. The ZF axioms on which the whole of mathematics can be built, cannot be proved to be consistent, but it can be proved to be incomplete.

So it is possible that a contradiction could be deduced from these axioms. But if, as we hope, they *are* consistent, they are certainly incomplete. There are truths about arithmetic (though not ones we'd be ever likely to meet) that can't be proved from any finite set of axioms! Mathematics is very far from being cut and dried.

At the heart of Gödel's proof is a very clever method for converting statements *about* the system into arithmetic statements *within* the system. For a start, statements are expressed symbolically, such as:

$$\forall x(-(x = 0) \rightarrow \exists y(xy = 1))$$

which means "for all $x$, if $x$ is not equal to zero then there exists $y$ such that $x$ times $y$ is equal to 1".

Gödel devised a system for coding these statements as a number by assigning a code to each symbol and building up a number for each statement on the basis of that. So, given a number $n$ one could, if that $n$ indeed represented a statement, decode it and so obtain the corresponding statement G($n$). Every possible statement would have a code, but not every code would correspond to a valid statement.

The numbers involved would be extremely large, but as this is an ‘in principle’ exercise, that isn’t a worry.

Now consider the statement that a given statement S is provable. A proof is just a list of statements, where each one is an axiom, or a previously proved theorem, or a logical consequence of the previous ones, and where the statement of the theorem is the last in the list. There’s a mechanical way of testing the validity of a proof and so one could, in principle, write a computer program for testing whether a given statement is provable from the axioms. It would be a case of generating all possible lists of statements that have S as the last statement, and then testing the ‘proof’ for validity.

Gödel showed how provability could be expressed as an arithmetic statement about natural numbers and so the statement P(*n*) = ‘the statement with Gödel number *n* is provable’ can be expressed as an arithmetic statement and so will have a certain Gödel number. Similarly, the statement N(*n*) = ‘the statement with Gödel number *n* is not provable’ has a Gödel number, say *g*.

Gödel then asked whether N(*g*) is true or false. The statement N(*g*) claims that it, itself, is unprovable. Thus we can obtain, as a purely arithmetic statement within the language of arithmetic, a statement which claims “I am unprovable”.

Now such a statement can't be false because being false would mean it was provable and hence true. It must therefore be true and hence it's a true but unprovable statement in arithmetic. But wait! Haven't we just proved that it's true?

Certainly we gave a meta-mathematical proof. But this proof is not one which could be expressed as an arithmetic proof within the system. Our unprovable statement is not unprovable in any absolute sense. It might not even be meaningful to talk about absolute unprovability.

N($g$) is unprovable in the relative sense that no proof of it could ever be constructed which starts from the axioms and proceeds using the rules of inference. And even if the axioms and rules were supplemented by others, so long as they remained finite in number, the existence of unprovable statements would remain.

# JOKE: PALINDROME

An Englishman, an Irishman and a Scotsman go into a bar. An American, who was already in the bar comes up to the Englishman and says, "Hey buddie, if you can tell me a good joke, I'll buy you a beer".

So the Englishman clears his throat and says, "37". At this the bar erupts into an uproar of laughter. The American looks puzzled, but says, "well it appears that was a great joke, so what'll you have?"

A little later the American goes up to the Scotsman and says, "I'll buy you a whisky if you can beat that last joke". The Scotsman stands on a stool, adjusts his kilt and says, "42". Once again the bar erupts into laughter, even louder than before. Several patrons are so carried away by their laughter that they roll around on the floor. So the American buys the Scotsman a Scotch.

A little later the American turns to the Irishman and says, "You Irish are renowned for your wonderful humour. I bet you can top that last joke – if you do, I'll buy you a pint of Guinness".

So the Irishman jumps up on the bar, adjusts his cap, and says, "93". There's deathly silence. Not even a murmur is heard. The American looks puzzled.

"I've worked out that you folks must number your jokes so that all you have to do is to give the joke number and you all know what the joke is. But back in the States we tell our jokes in full. Now I'm a little puzzled why that last joke fell so flat. What went wrong?"

“Ah”, says the Scotsman, “you know what the Irish are like. They’re always getting things back to front.”

“Well”, said the American, “would you mind telling me that last joke in full”.

“Och, aye”, said the Scotsman, “but are ye sure ye want to hear it. As I said it’s not very funny”.

“Well, yes”, said the American, “I’m fascinated by British humour”.

“OK”, says the Scotsman, “Joke number 93 goes like this. An Englishman, an Irishman and a Scotsman go into a bar. An American, who was already in the bar comes up to the Englishman and says, “Hey buddie, if you can tell me a good joke I’ll buy you a beer”. So the Englishman clears his throat and says, “37”.